Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NOTE ON THE PRODUCT OF LINEAR SUBSTITUTIONS.*

By H. B. Newson.

If two linear substitutions in n variables be compounded, the product is also a linear substitution in n variables. The following method of expressing the result in determinant form is believed to be new. The proof is given for two substitutions in three variables, but the method and result are capable of immediate generalization for n variables.

Let T and T_1 be two substitutions as follows:

$$T: \quad \begin{aligned} \rho x_1 &= a_1 x + b_1 y + c_1 z, \\ \rho y_1 &= a_2 x + b_2 y + c_2 z, \\ \rho z_1 &= a_3 x + b_3 y + c_3 z, \end{aligned} \qquad T_1: \quad \begin{aligned} \rho_1 x_2 &= a_1 x_1 + \beta_1 y_1 + \gamma_1 z_1, \\ \rho_1 y_2 &= a_2 x_1 + \beta_2 y_1 + \gamma_2 z_1, \\ \rho_1 z_2 &= a_3 x_1 + \beta_3 y_1 + \gamma_3 z_1. \end{aligned}$$

The substitution T_2 is obtained by eliminating x_1, y_1, z_1 from the above equations. This may be done as follows: Find the inverse of T by solving the three equations of T for x, y, z, Thus we get

$$T^{-1}: \quad \begin{aligned} \frac{\Delta}{\rho} x &= A_1 x_1 + A_2 y_1 + A_3 z_1, \\ \frac{\Delta}{\rho} y &= B_1 x_1 + B_2 y_1 + B_3 z_1, \\ \frac{\Delta}{\rho} z &= C_1 x_1 + C_2 y_1 + C_3 z_1, \end{aligned}$$

where Δ is the determinant of T and A, B, etc. have the usual meanings.

The three equations of T^{-1} and the first one of T_1 form a system of four simultaneous linear equations; hence

$$\begin{vmatrix} -\frac{\Delta}{\rho} x & A_1 & A_2 & A_3 \\ -\frac{\Delta}{\rho} y & B_1 & B_2 & B_3 \\ -\frac{\Delta}{\rho} z & C_1 & C_2 & C_3 \\ -\rho_1 x_2 & a_1 & \beta_1 & \gamma_1 \end{vmatrix} = 0.$$

* Read before the Chicago Section of the American Mathematical Society at the Evanston meeting, 2/3 January, 1902.

This equation expresses the relation between x, y, z and x_2. Solving the last equation for x_2 we get

$$\rho\rho_1\Delta x_2 = \begin{vmatrix} x & A_1 & A_2 & A_3 \\ y & B_1 & B_2 & B_3 \\ z & C_1 & C_2 & C_3 \\ 0 & \alpha_1 & \beta_1 & \gamma_1 \end{vmatrix}.$$

In like manner we get similar results for y_2 and z_2; thus

$$\rho\rho_1\Delta y_2 = \begin{vmatrix} x & A_1 & A_2 & A_3 \\ y & B_1 & B_2 & B_3 \\ z & C_1 & C_2 & C_3 \\ 0 & \alpha_2 & \beta_2 & \gamma_2 \end{vmatrix}; \qquad \rho\rho_1\Delta z_2 = \begin{vmatrix} x & A_1 & A_2 & A_3 \\ y & B_1 & B_2 & B_3 \\ z & C_1 & C_2 & C_3 \\ 0 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}.$$

When these three determinants are expanded, Δ divides out of both sides of the equation.

The general formula for n variables is

$$\rho\rho_1\Delta^{n-2}x_i''' = \begin{vmatrix} x_1 & A_1 & A_2 & \cdots & A_n \\ x_2 & B_1 & B_2 & \cdots & B_n \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ x_n & N_1 & N_2 & \cdots & N_n \\ 0 & \alpha_i & \beta_i & \cdots & \nu_i \end{vmatrix} (i = 1 \cdots n).$$

THEOREM. *The value of x_i''' in the product of T and T_1, two linear substitutions, is proportional to the determinant formed by bordering the determinant of T^{-1}, the inverse of T, vertically by the variables of T and horizontally by the coefficients of the ith equation in T_1.*

UNIVERSITY OF KANSAS.
LAWRENCE, KANSAS.